

Chapter X

Uncertainty Visualization

The Prepress Version

While uncertainty is present in most data analysis pipelines, reasoning with uncertainty is challenging for novices and experts alike. Fortunately, researchers are making significant advancements in the communication of uncertainty. In this chapter, we detail new visualization methods and emerging cognitive theories that describe how we reason with visual representations of uncertainty. We describe the best practices in uncertainty visualization and the psychology behind how each approach supports viewers' judgments. This chapter begins with a brief overview of conventional and state-of-the-art uncertainty visualization techniques. Then we take an in-depth look at the pros and cons of each technique using cognitive theories that describe why and how the mind processes different types of uncertainty information.

1.1 Introduction

Uncertainty is inherent to most data and can enter the analysis pipeline during the measurement, modeling, and forecasting phases [1]. Effectively communicating uncertainty is necessary for establishing scientific transparency. Further, people commonly assume that there is uncertainty in data analysis, and they need to know the nature of the uncertainty to make informed decisions [2]. However, understanding even the most conventional communications of uncertainty is highly challenging for novices and experts alike [3], which is due in part to the abstract nature of probability and ineffective communication techniques. Reasoning with uncertainty is unilaterally difficult, but researchers are revealing how some types of visualizations can improve decision-making in a variety of diverse contexts, from hazard forecasting [4, 5] to healthcare communication [6], to everyday decisions about transit [7].

Scholars have distinguished different types of uncertainty, including aleatoric (irreducible randomness inherent in a process), epistemic (uncertainty from a lack of knowledge that could theoretically be reduced given more information), and ontological uncertainty (uncertainty about how accurately the modeling describes reality, which can only be described subjectively) [8]. The term risk is also used in some decision-making fields to refer to quantified forms of aleatoric and epistemic uncertainty, whereas uncertainty is reserved for potential error or bias that remains unquantified. Here we use the term uncertainty to refer to quantified uncertainty that can be visualized, most commonly a probability distribution.

This chapter begins with a brief overview of the common uncertainty visualization techniques and then elaborates on the cognitive theories that describe how the approaches influence judgments. The goal is to provide readers with the necessary theoretical infrastructure to critically evaluate the various visualization techniques in the context of their own audience and design constraints. Importantly, there is no one-size-fits-all uncertainty visualization approach guaranteed to improve decisions in all domains, nor even guarantees that presenting uncertainty to readers will necessarily improve judgments or trust. Therefore, visualization designers must think carefully about each of their design choices or risk adding more confusion to an already difficult decision process.

Uncertainty Visualization Design Space

There are two broad categories of uncertainty visualization techniques, as shown in Figure 1.1. The first are graphical annotations that can be used to show properties of a distribution, such as the mean, confidence/credible intervals, and distributional moments. Numerous visualization techniques use the composition of marks (i.e., geometric primitives,

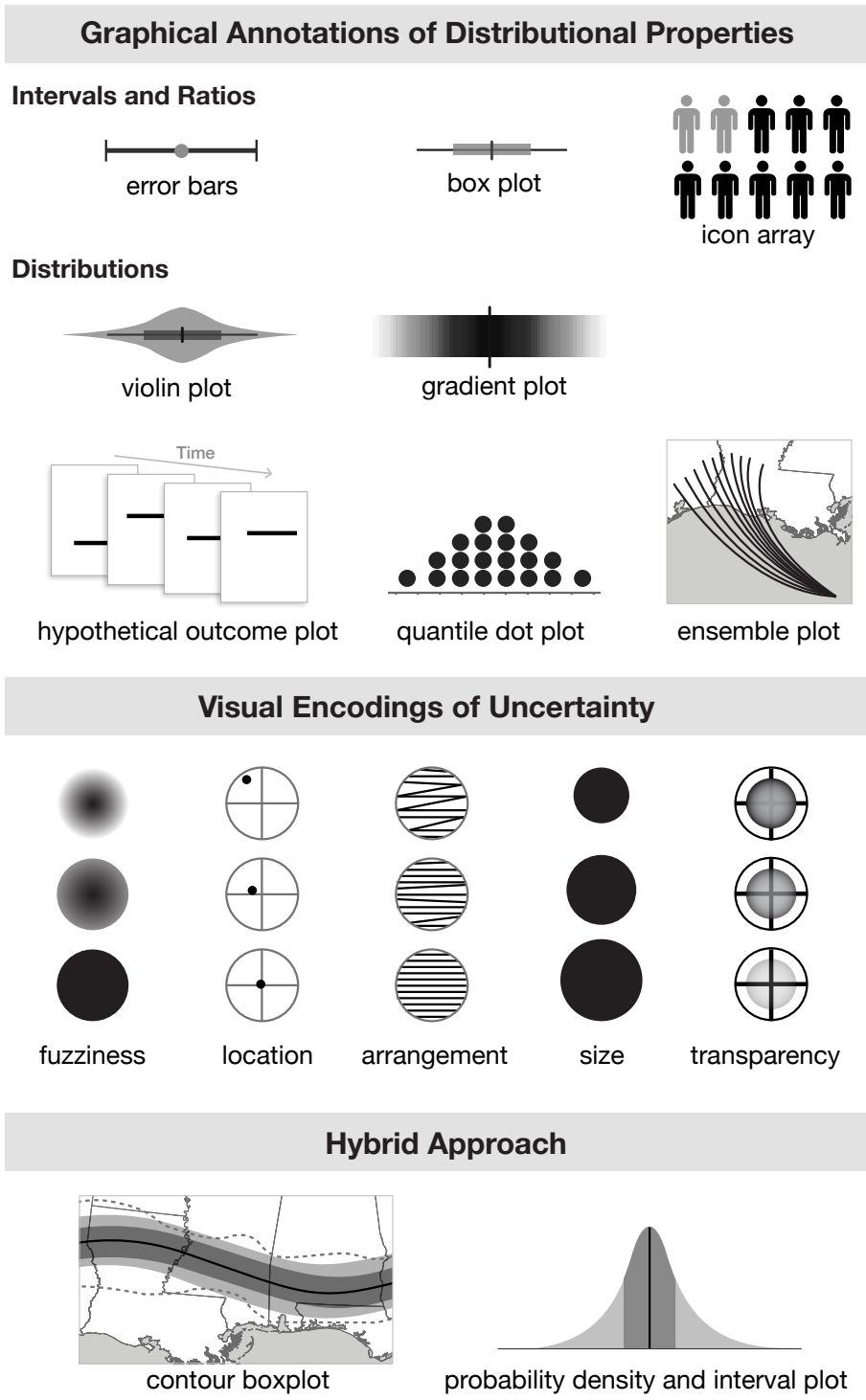


Figure 1.1: A subset of the graphical annotations used to show properties of a distribution and mappings of probability/confidence to visual variables. The visual variables that require color printing were excluded (e.g., color hue, color value, and color saturation). The examples are adapted from prior work: violin and gradient plots [9], hypothetical outcome plots [10], quantile dotplot [11], ensemble plot [12], icon array [13], fuzziness transparency [14], contour boxplot [15], and probability density and interval plot [7].

such as dots, lines, and icons [16]) to display uncertainty directly, as in error bars depicting confidence or credible intervals. Other approaches use marks to display uncertainty implicitly as an inherent property of the visualization [17]. For example, hypothetical outcome plots (HOPs) [10] are random draws from a distribution that are presented in an animated sequence, allowing viewers to form an intuitive impression of the uncertainty as they watch. The second category of techniques focuses on mapping probability or confidence to a visual encoding channel (for reviews, see [14, 18, 19]). Visual encoding channels define the appearance of marks using controls such as color, position, and transparency [16]. Techniques that use encoding channels have the added benefit of adjusting a mark that is already in use, such as making a mark more transparent if the uncertainty is high. Marks and encodings that both communicate uncertainty can be combined to create hybrid approaches, such as in contour box plots [20] and probability density and interval plots [7] (see Figure 1.1). In Figure 1.1, the contour box plot shows 95% intervals with greater transparency than 50% intervals and includes a mean line (black line) and outliers (dotted lines). Also, the probability density and interval plot in Figure 1.1 shows the shape of a density function with 50% intervals in darker gray and a mean line in black.

Some of the most common techniques in scientific communication are those that display intervals (see Figure 1.1, error bars and box plots). Despite their everyday use, visualizations of intervals have widely documented issues (e.g., [3, 9, 21]), such as provoking viewers to incorrectly think of distributional data as categorical [22]. For example, when summary information about the location of a natural disaster is plotted on a map with a contour line, people incorrectly interpret the area within the contour as the danger zone and locations just outside as safe [21] (see Section 1.2 Visual Boundaries = Cognitive Categories). Visualizations of intervals are generally hard for both experts and novices to use [3], and errors persist even with extensive instructions [23]. Rather than visualizing intervals, some research finds that using more expressive visualization techniques (e.g., violin and gradient plots in Figure 1.1 [9]) can help people understand the uncertainty in the data more effectively. More expressive visualizations provide a fuller picture of the data by depicting more properties, such as the nature of the distribution and outliers, which can be lost with intervals.

Other work proposes that showing distributional information in a frequency format (e.g., 1 out of 10 rather than 10%) more naturally matches how people think about uncertainty and can improve performance (e.g., quantile dotplot and icon arrays in Figure 1.1 [11, 13], see Section 1.2 Frequency Framing). Visualizations that represent frequencies tend to be highly effective communication tools, particularly for individuals with low numeracy (e.g., inability to work with numbers) [24], and can help people overcome various decision-making biases [6]. Some approaches even require viewers to account for the uncertainty in making judgments of summary statistics (e.g., HOPs) [10], which can be useful because uncertainty information is commonly ignored or mentally substituted for simpler information (see Section 1.2 Attribute Substitution).

Researchers have dedicated a significant amount of work to examining which visual encodings are most appropriate for communicating uncertainty, notably in geographic information systems and cartography [14, 18, 19] (see Visual Encodings in Figure 1.1 and Section 1.2 Visual Semiotics). One goal of these approaches is to evoke a sensation of uncertainty, for example, using fuzziness, fogginess, or blur. Other work that examines uncertainty encodings also seeks to make looking-up values more difficult when the uncertainty is high, such as value-suppressing color palettes [25]. Given that there is no one-size-fits-all technique, in the following sections, we detail the emerging cognitive theories that describe how and why each visualization technique functions.

1.2 Uncertainty Visualization Theories

The empirical evaluation of uncertainty visualizations is challenging [26]. Many user experience goals (e.g., memorability [27], engagement, and enjoyment [28]) and performance metrics (e.g., speed, accuracy, and cognitive load [29]) can be considered when evaluating uncertainty visualizations [26]. Beyond identifying the metrics of evaluation, even the most simple tasks have countless configurations. As a result, it is hard for any single study to sufficiently test the effects of a visualization to ensure that it is appropriate to use in all cases. Visualization guidelines based on a single or small set of studies are potentially incomplete. Theories can help bridge the gap between visualization studies by identifying and synthesizing converging evidence, with the goal of helping scientists make predictions about how a visualization will be used. Understanding foundational theoretical frameworks will empower designers to think critically about the design constraints in their work and generate optimal solutions for their unique applications. The theories detailed in the next sections are only those that have mounting support from numerous evidence-based studies in various contexts. As an overview, Table 1 provides a summary of the dominant theories in uncertainty visualization, along with proposed visualization techniques.

Theory	Summary	Visualization Techniques
Frequency Framing [30] (Section 1.2)	Uncertainty is more intuitively understood in a frequency framing (1 out of 10) than in a probabilistic framing (10%)	icon array [13], quantile dotplot [11], hypothetical outcome plots [10]
Attribute Substitution [31] - Deterministic Construal Error [32] (Section 1.2)	If given the opportunity, viewers will mentally substitute uncertainty information for data that are easier to understand	hypothetical outcome plots [10]
Visual Boundaries = Cognitive Categories [21] (Section 1.2)	Ranges that are represented by boundaries lead people to believe that data inside and outside the boundary are categorically different	ensemble display [12], error bar alternatives [7, 9]
Visual Semiotics [14] (Section 1.2)	Some encoding techniques naturally map onto uncertainty	fuzziness, transparency, location, etc. [14], value-suppressing color pallet [25]

Table 1.1: Summary of uncertainty visualization theory detailed in this chapter.

Frequency Framing

The frequency framing hypothesis was initially proposed by Gerd Gigerenzer [30] in response to popular theories, which argued that human reasoning systematically deviates from rational choice according to mathematical rules (e.g., [33]). Gigerenzer hypothesized that our decisions seem flawed when we are provided with confusing information, such as probabilities communicated as percentiles (e.g., 10% chance). However, individuals can make rational choices if provided with information in a format they can understand easily, such as in frequencies or ratios (e.g., 1 out of 10). Gigerenzer argued that percentiles do not match the way people encounter probability in the world, and therefore lead to errors. Instead, it is more intuitive to depict probability as a frequency, as we have more exposure to these types of ratios (e.g., I hit traffic on this road 7 out of 10 times. I will take a different route tomorrow.) The frequentist framing hypothesis has substantial support from studies that find we can relatively automatically and accurately understand frequency formats, whereas probabilities are time consuming and highly error prone (for review and caveats, see [34]).

One of the most effective ways to implement frequency framing of uncertainty information is with visualizations, and in this section we will detail two promising frequency-framing techniques. Researchers, predominantly in health-care communication, have extensively studied the use of icon arrays (see Figure 1.1) to display ratios and have found strong evidence that they are useful for communicating forecasted probabilities of event outcomes. The second notable use of frequency formats in visualization is within the emerging study of quantile dotplots (see Figure 1.1). Whereas quantile dotplots are relatively new and have not received as much examination as icon arrays, they capitalize on the theoretical benefits of frequency framing and have demonstrated positive results in laboratory studies.

Icon arrays

A substantial body of research demonstrates that icon arrays are one of the most effective ways to communicate a single probabilistic value and can outperform textual descriptions of probabilities and frequencies [27, 35–42]. One of the key benefits of icon arrays is that they offload cognition by allowing a viewers visual system to compare the denominator and the numerator in a frequency probability format. Visual comparisons of this nature are easier and faster than numerical calculations.

The difficulty in comparing ratios can produce common errors, such as individuals focusing on the numerator of each ratio and neglecting the denominator, called denominator neglect (for review see [43]). For example, when comparing a cancer with a mortality rate of 1,286 of 10,000 people to a cancer with a mortality rate of 24 of 100 people, participants in a laboratory study incorrectly reported that the former cancer was riskier [44]. Researchers propose that individuals pay more attention to the relative differences in numerators (in this case, 1,286 vs. 24 deaths), even though they should consider the relative ratios (12.86% vs. 24% mortality) (e.g., [43, 44]). Several studies

have found that icon arrays can reduce denominator neglect by allowing people to compare relative ratios visually (e.g., [13, 42, 45, 46]). Additionally, other studies have found that people trust icon arrays more than other common visualization techniques [35], and they can reduce decision-making biases, including anecdotal evidence bias [27], side effect aversion [38, 47], and risk aversion [48].

The positive impacts of icon arrays, particularly on medical decision-making, are relatively consistent across studies that use various types of icons. However, if designers are interested in optimizing their icon selections, they should consider showing part-to-whole comparisons (i.e., both the denominator and the numerator). Designers should avoid showing only the numerator with icons and adding the denominator in text because viewers will make their judgments by considering the numerator and ignoring the denominator [46]. Icon arrays function by directing the viewers attention to the information in the icons, so all the relevant information must be shown. Further, it is important to arrange the icons systematically in a grid that is easy to count. Various studies have found that icon arrays that are not arranged systematically are challenging to use [37], particularly for those with low numeracy [49, 50]. If two or more arrays will be compared, they should use the same denominator for each array, which will make the comparison easier.

Quantile dotplots

Icon arrays may be useful for communicating discrete data where only a small number of outcomes are possible (e.g., a positive or negative test result) [51]. When it comes to visualizing continuous variables, common approaches include probability density plots, which map probability to height (and by extension, area). However, users may have difficulty determining the exact density for any value because they need to visually calculate the integral under the curve [11]. Kay et al. [11] created the quantile dotplot as a frequency-framed alternative for displaying uncertainty for a continuous variable. As seen in Figure 1.2, a quantile dotplot represents a distribution where dots are sampled proportional to the quantiles of the distribution. In this case, each dot depicts a 5% probability. Using this figure as an illustration, imagine that the viewers task is to determine if a bus will arrive in 8 mins or later. With the quantile dotplot, the viewer can count the dots to determine that there is a 90% chance the bus will arrive 8 mins or later.

Quantile dotplots have been tested in several empirical studies, which have found that they reduce the variance of probabilistic estimates compared to density plots [11] and improve recall of distributional data [52]. Other studies have found that quantile dotplots are more useful for decisions with risk compared to interval and density plots, and are significantly better than textural descriptions of uncertainty [7]. Figure 1.2 illustrates the process of generating a quantile dotplot from a log-normal distribution.

Note that another way people can interpret both quantile dotplots and icon arrays is to make a visual area judgment. If viewers were to make an area judgment, they would not be utilizing the frequency information. Hence, icon arrays and quantile dotplots support both frequency- and non-frequency-based inferences. HOPs (see Figure 1.3) are another example of visualizations that can use frequency framing in a way that viewers cannot fall back on non-frequency-based inferences. HOPS are described in the following section (Attribution Substitution Section 1.2) because they have the added benefit of requiring the viewer to consider uncertainty, which is an archetypal example of the theory detailed in that section.

Attribute Substitution

Reasoning with uncertainty is classically challenging, and one strategy that people unconsciously use to deal with difficult information is substitution [31]. Individuals will substitute a hard mental computation for an easier one. Researchers have studied this process extensively and termed it the attribute substitution heuristic [31]. A heuristic is a rule of thumb that people use to make decisions quickly, which can be beneficial if the heuristic produces a correct judgment or detrimental [53], as is the case with the deterministic construal error in visualizations [32].

The deterministic construal error is when individuals attempt to substitute visual uncertainty information for deterministic information. For example, Joslyn and LeClerc [32] found that when participants viewed mean temperature forecasts that included 95% confidence intervals depicted as bars with end caps, they incorrectly believed that the error bars represented high and low temperatures. The participants maintained this belief even when Joslyn and LeClerc tested a condition where the correct way to interpret the forecast was shown prominently in a key to the side of the display [32]. The authors proposed that viewers were substituting the complex uncertainty information for high and low temperature forecasts that were easier to understand [22, 32]. Other research with static visualizations has reported similar findings in which, if given the opportunity, viewers interpret uncertainty information incorrectly as deterministic information [3, 54, 55].

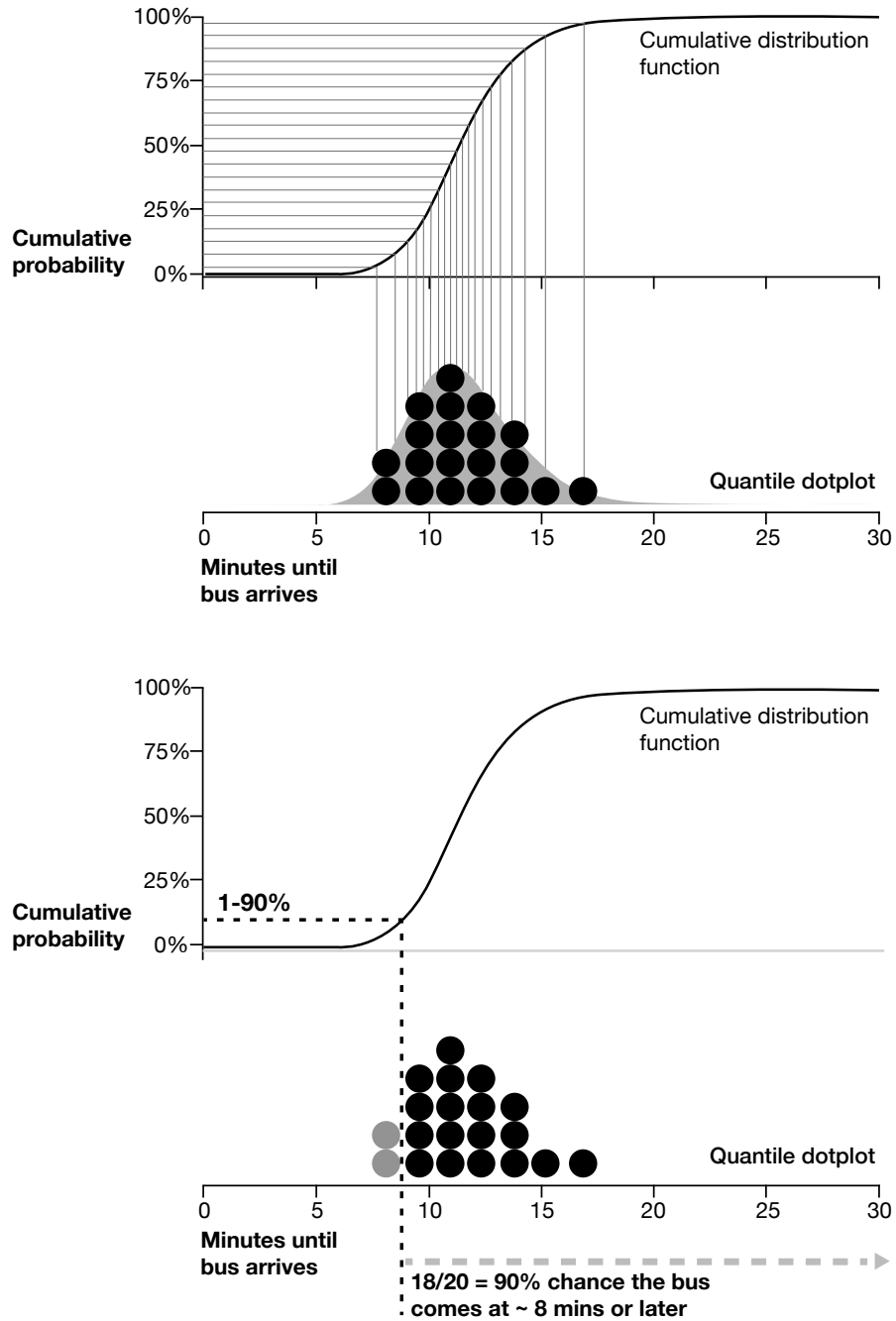


Figure 1.2: Based on figure and description from Kay et al. [11]. Tutorial in R can be found at <https://github.com/mjskay/when-ish-is-my-bus/blob/master/quantile-dotplots.md>

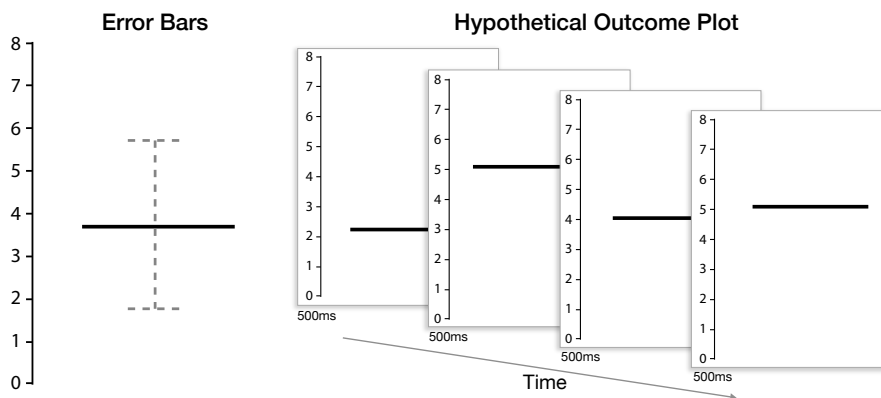


Figure 1.3: Illustration of HOPs compared to error bars from the same distribution, based on a figure from Hullman, Resnick, and Adar [10].

Note that such a deterministic construal error has been found with visualizations and not in textual descriptions of the same information [32], meaning that the deterministic construal error might be a visual-spatial bias, which is a bias that is produced by the visual system [21]. In a paper that illustrates a cognitive model of decision-making with visualizations, Padilla et al. [21] proposed that visual-spatial biases are a unique form of bias that occurs early in the decision-making process (during visual encoding), making these type of biases particularly challenging to overcome as they influence all the downstream processes. Emerging research supports this hypothesis by demonstrating that even with extensive instructions, viewers judgments are still influenced by deterministic construal errors, even though they are able to report the correct strategy at the end of the study [4, 23].

Hypothetical outcome plots

When viewers may be likely to ignore uncertainty in favor of simpler heuristics, any static visualization that encodes summary statistics runs the risk of allowing them to discount uncertainty in their judgments. One promising approach that can help to reduce the discounting of uncertainty is hypothetical outcome plots (HOPs; [10]). HOPs use random draws from a distribution and animate the draws over time. Figure 1.3 presents an example set of frames in which each frame is one random pull from the distribution on the left. The frames are shown in a random sequence for a short time (i.e., <500ms), which creates an animation that can give viewers an intuitive sense of the uncertainty in the true mean.

A crucial theoretical contribution of HOPs is that they require viewers to build up a representation of distributional information in their mind. This approach has no one outcome that viewers can fixate on. Instead, they are forced by the visualization technique to both 1) account for uncertainty in their understanding of the data, and 2) recognize that less probable outcomes do fall within the distribution. The second point is vital for hazard forecasting, where members of the public may be upset when a less likely event occurs because they failed to understand the full range of forecasted outcomes (e.g., Hurricane Katrina or the L'Aquila earthquake [56]). Empirical studies provide some support for the benefits of HOPs for lay viewers, finding that they can outperform static error bars [10, 57], icon arrays [58], line ensembles [57], and violin plots [10].

Another crucial aspect of HOPs compared to static uncertainty displays is that they can be applied to most distribution data and visual encoding techniques relatively easily, as long as animation is not already used. To create HOPs, one must first be able to draw samples from the distribution of interest, whether univariate (as in Figure 3) or multivariate. The draws can be generated via bootstrapping, referring to the large family of statistical techniques, which are appropriate for numerous data types. For example, bootstrapping can be used for generating hypothetical samples from an observed dataset, including parametric approaches (i.e., a model is fit to observed data and then samples are drawn from the model), and non-parametric approaches (i.e., resampling with replacement from observed data, which relaxes distributional assumptions). A large number of these samples can then be animated in sequence, with each sample appearing for only a short period of time. Research suggests that frame rates of 400 to 500ms tend to perform best. HOPs are particularly useful in the case of complex visualizations where the distribution to be conveyed is a joint distribution with potential dependencies between variables. When visualizations already use the most accurate visual properties to show the data, for example, a choropleth map that uses position to show geographic location and color to

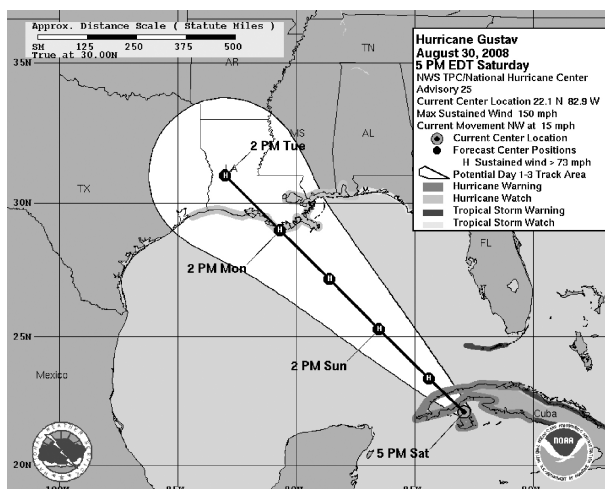


Figure 1.4: Example Cone of Uncertainty produced by the National Hurricane Center, originally printed in Padilla et al. [5], reprinted with permission.

show the value of a variable, conveying uncertainty may be difficult because it requires adding another visual property to an already complex visualization. As long as a visualization is not already animated, HOPs can be used without requiring the designer to choose another encoding for uncertainty, and naturally display joint probabilities. This ability has inspired visualization researchers to use probabilistic animation to show uncertainty in geospatial data [59], as well as complex visualizations like parallel coordinates plots [60].

Visual Boundaries = Cognitive Categories

Padilla et al. [21] proposed the data visualization theory that when visual boundaries, such as isocontours and error bars, are used for continuous data, the boundaries lead people to conceptualize the data as categorical. This theory is based on work by Barbra Tversky [61], in which she proposed that visual-spatial communications are intrinsically related to thought. To illustrate how concepts like containment influence how we understand visual information, she writes, Framing a picture is a way of saying that what is inside the picture has a different status from what is outside the picture (p. 522 [61]). Padilla et al. [62] demonstrated how people perform the same task differently when presented with continuous or categorical depictions of the same data. The authors found that, in some cases, binned 2D scalar fields produce faster and more accurate judgments than continuous encodings of the same data, which may lead some individuals to conclude that they should always bin continuous data. However, the theory proposed by Padilla et al. [21] (visual boundaries = cognitive categories) emphasizes that visual boundaries force people to think differently about the data than a continuous encoding, which can be good or bad depending on the nature of their decision.

The issue for uncertainty visualization is that most uncertainty data types are continuous. When a designer processes uncertainty data into a categorical format (e.g., mean values, ranges, or intervals), it fundamentally changes the way that a user forms an understanding of the data. For example, in hurricane forecasting, the most common way to represent the uncertainty in the storm's path is with the Cone of Uncertainty (see Figure 1.4). The Cone of Uncertainty is the current method used by the National Hurricane Center, and the border of the cone represents a 66% confidence interval around the mean predicted path. In numerous studies, researchers have found that viewers believe areas inside the cone are categorically different from areas outside the cone [5, 23, 55]. When visualized with boundaries, viewers cannot ascertain that there is a distribution of uncertainty in the storm's path. The cognitive category created by the border of the cone makes viewers believe that areas inside the cone are in the danger zone, and areas outside are relatively safe [5, 55]. Participants' decisions are influenced by a subconscious categorical interpretation of the cone even when they are given instructions about how to interpret the visualization correctly and they can report the correct judgments at the end of the study [23]. The result of this inaccurate and persistent categorization may cause some people who reside just outside the cone to believe they are safe and not take preparatory actions.

Other researchers also have found evidence that users conceptualize areas inside a boundary differently than areas outside [63, 64]. For example, McKenzie et al. [63] examined how users make decisions about the positional uncertainty in their location using Google Map's blue dot visualization. When the authors presented viewers with a version

of Google Map's blue dot with hard boundaries, individuals judgments were based on determining if a location was inside or outside the boundary [63]. Newman and Scholl [64] also demonstrated how boundaries produce categorization with bar charts. Participants in the Newman and Scholl study were shown mean values with bar charts and asked to make judgments about if a data point was likely from the population depicted by the bar. If the data point fell within the bar, the participants were more likely to believe that it came from the represented population. However, they believed that data points that were the same distance from the mean but located just outside the bar were from a different population [64]. The authors proposed that this within-the-bar-bias is due to perceptual object grouping, where our visual system groups items that are located near one another. The theory proposed in Padilla et al. [21] additionally suggests that our cognitive systems attempt to organize information in the world by grouping information into cognitive categories, and that this process is not purely a function of the visual system.

Cognitive categories have no inherent problem, and in some cases, designers might want their viewers to think about data categorically [62]. The concern for uncertainty visualization is that sometimes the boundaries are not well considered, and different choices about which boundaries to show result in different judgments. For the Cone of Uncertainty, in particular, there is no longer a justification for why the boundary is located at 66% (i.e., Why not 95% or 75%?). By plotting a hard boundary, viewers assume that the scientists are suggesting that the specific value of the boundary is important. Viewers understandably assume the value of a boundary is meaningful, particularly when the information about how the visualization was generated is insufficient, which is the case with hurricane news forecasts. In an analysis of the 20 most viewed television forecasts for Hurricane Irma in 2017, Padilla et al. [4] found that zero newscasters detailed how the Cone of Uncertainty was created or how to interpret it correctly, and the average length of the forecast was merely 1:52 minutes. Viewers have no choice but to assume that the scientists who made the forecast are indicating an important distinction with the boundary of the cone.

Ensemble displays

There are several alternatives to interval displays, such as the previously detailed HOPs. However, animations are not feasible in some cases. For example, in hurricane forecasting, static visualizations may be needed for printed reports or for regions that might not have access to high-speed Internet. Further, for hurricanes and other hazards, the time course of the hazard is uncertain. It is possible that when viewing HOPs of data where time information is critical, such as a natural disaster, viewers may incorrectly assume the animation is depicting an event unfolding over time. Ensemble displays (see Figure 1.5) are another alternative to summary visualizations that researchers have tested extensively in the context of hurricane forecasting [4, 5, 12, 55, 65]. Ensemble displays are traditionally generated by making perturbations to a models parameters and plotting the resulting runs on a static display [12]. The result is a visualization that intrinsically shows the uncertainty in the storm's path. Early versions of the ensemble display outperformed the Cone of Uncertainty and other visualization techniques of the storm's path in laboratory studies [5, 12, 55].

Ensemble hurricane paths have received some criticism because older versions can look confusing, like a plate of spaghetti (a.k.a., spaghetti plots, see Figure 1.5 right). Researchers have addressed concerns by developing a method for reconstructing the predicted paths from runs of the model [65] (see Figure 1.5 left). The paths of the reconstructed ensembles have good separation and still communicate the uncertainty in the trajectory of the storm. The other benefit of using a path reconstruction procedure is that fewer lines are needed to show the full range of outcomes, which declutters the display. In the case of hurricane forecasting, decluttering the display allows researchers to make the paths thicker and represent the intensity of the storm in color and the size of the storm with a glyph. Liu et al. [65] found that their study participants could effectively make decisions that incorporated the path, size, and intensity of the storm when it was visualized as an ensemble display.

Ensembles outperform all other versions of hurricane path visualizations, but they have also received significant alternative hypothesis testing by their creators, which has produced some caveats [4, 5]. In visualization research, alternative hypothesis testing is when researchers attempt to discover cases when their technique is not effective rather than focusing on optimal use cases. Researchers have found that people overreact when they see one ensemble member impacting their point of interest, such as their town [4, 5]. The same people do not overreact when an ensemble member barely misses their point of interest. Further, this effect is influenced by the number of ensemble members shown. For example, people overreact more when 1 out of 9 ensembles appear to be hitting their point of interest compared to 1 out of 33. This effect can be reduced only partially with training on how to interpret the displays correctly [4].

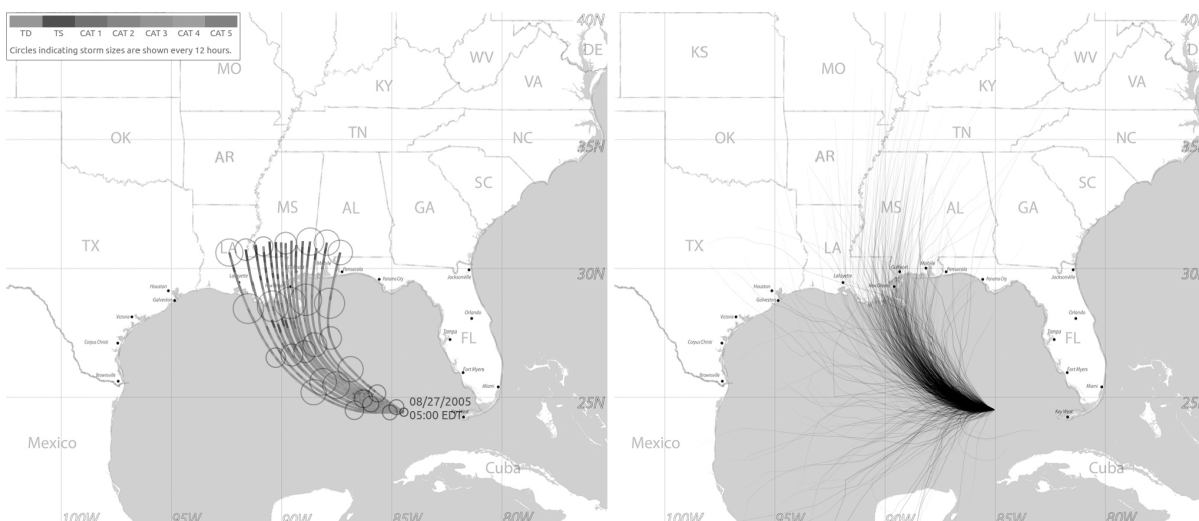


Figure 1.5: Left is an example of an ensemble hurricane path display that utilizes a path reconstruction procedure detailed in Liu et al. [65] and that also shows the intensity of the storm in the path color and the size of the storm with circle glyphs. Right is an earlier version of the ensemble display examined in [5, 12, 55] that does not use the path reconstruction procedure.

Error bars

In cases where researchers are interested in categorical interpretations, the summary statistics should be considered because of the difficulty we have interpreting sampling distributions of the mean shown as confidence intervals or standard error intervals [52, 66]. The sampling distribution is the distribution of means expected if one were to repeatedly draw samples of a given size n for a population. For example, when viewing results of an evaluation of a new drug relative to a control, one might wonder how much taking a new drug is likely to help a randomly drawn patient. Recent work has shown that when error bars are used to denote a standard error range of a control and treatment effect, laypeople are willing to (over)pay more for the treatment and overestimate the size of the effect compared to when the error bars show a standard deviation range [67]. Further, the relationship between statistical significance and whether or not two error bars overlap is often misunderstood: when two frequentist 95% confidence interval error bars do not overlap, it is correct to assume that the difference between the two quantities is significant at an alpha level of 0.05. However, when the two intervals do not overlap, it is incorrect to assume, as even researchers have been shown to do [3], that the difference between the two quantities is not significant.

In contrast to denoting uncertainty through boundary graphical marks meant to separately encode variance from central tendency and other distributional moments, approaches that map probability to a visual variable make uncertainty intrinsic to the presentation of other properties of a distribution like the mean. Correll and Gleicher [9] found that violin plots (mapping probability to area or width at a given y position) and gradient plots (mapping probability to opacity) lead to more intuitive assessments of value likelihood and ability to surprise, which are more closely aligned with statistical definitions of uncertainty. However, the findings of other studies are less clear regarding how violin and density plots compare to error bars. Hullman et al. [10] observed little difference between judgments about probabilities from single distributions and multiple distributions from violin plots and error bars showing a standard deviation range. Fernandes et al. [7] found that a density plot leads to better quality decisions in a transportation context than in an interval, but users who used a hybrid density plot with overlaid Bayesian 50% and 95% credible intervals made better decisions after practice with the display than users of either encoding in isolation.

Visual Semiotics of Uncertainty

The final theory we detail in this chapter is the proposal that uncertainty encoding techniques that utilize visual metaphors for uncertainty, such as graying out using color saturation [68], out of focus using blur [69], fogginess using transparency [70, 71], adding noise using texture [72] or sketchiness [73], are a more intuitive ways to communicate uncertainty (see Figure 1.1 for examples and [14, 18, 19] for excellent reviews). The theory of visual semiotics of uncertainty, proposed by MacEachren et al. [14], suggests that visual encodings that prompt appropriate metaphors

are easier to map onto the corresponding aspects of the information. This theory proposes that features that viewers spontaneously interpret as conveying uncertainty will be more effective than features that do not evoke uncertainty associations. For example, MacEachren et al. [14] asked viewers to judge the intuitiveness of numerous visual encodings of uncertainty (see Figure 1.1). They found that fuzziness, location, value, arrangement, size, and transparency were rated as highly intuitive. The theory of visual semiotics of uncertainty [14] has inspired numerous applications of metaphoric uncertainty visualization from cultural collections [74] to educational reporting systems [75].

In addition to the metaphoric association of uncertainty, some of the visualizations in this class of approaches map probability to visual properties and are designed to inhibit the viewer from resolving the value of a datum when the uncertainty is too high. For example, the location of a point on a map can be blurred proportional to the uncertainty in the position, such that the viewer cannot resolve an exact location [63]. Value-suppressing uncertainty palettes [25], which similarly attempt to make perception difficult in proportion to uncertainty, attempt to improve upon bivariate color maps that plot uncertainty to a separate color dimension from value, by making value judgments more difficult for uncertainty values. A viewer may be able to judge the value of a datum separately from its uncertainty using the bivariate color map by focusing only on hue, but the value-suppressing palette blends hues denoting value with gray proportional to how uncertain they are. As a result, the most uncertain values all appear as the same shade of gray. Correll, Moritz, and Heer [25] found that when applied to choropleth maps, users weight uncertainty more heavily using the value-suppressing palettes in a decision task compared to the bivariate color map. The two key contributions of these approaches are that they elicit metaphoric associations with uncertainty and they restrict viewers from making overly precise judgments when uncertainty is high. In the following paragraphs, we will discuss the implications of these contributions in turn.

The theory for why it is beneficial for uncertainty visualization to metaphorically depict uncertainty has to do with the concept of natural mappings [21, 28]. Natural mappings suggest that there are ways to display information that closely aligns with how people naturally think about the data. The importance of the alignment between an individual's mental representation of the data and the visual depiction of the data was initially described by Pinker [28] and expanded into a decision-making framework by Padilla et al. [21]. The theory suggests that when a visual representation matches how people think about the data, they will use their cognitive effort reserves to complete the task effectively. In contrast, if the discrepancy between how the information is presented and how people conceptualize it is large, they will first transform the visual variables in their minds to match their mental representation [28]. The transformation step uses some of the viewer's limited amount of mental effort, and less effort is left for the task. Uncertainty visualizations that naturally map onto how we conceptualize uncertainty may improve performance because viewers may not need to do superfluous mental transformations.

The theory of naturalness describes why metaphoric encodings of uncertainty may be helpful, but a variety of open questions concerning the exact nature of naturalness remain. Chief among them is the problem of determining how people conceptualize data. Without understanding a viewer's mental representation of data, attempts to naturally match how we think about data are guesses. Although educated guesses about how we mentally represent information are a good start, additional research is needed that more scientifically identifies our mental schemas for each type of data and context. Additionally, we have no clear way to determine the degree of relatedness between our conceptualization and the visual encoding. A range of relatedness likely impacts the extent of mental transformations required.

A more concrete contribution of metaphoric uncertainty encodings is that some techniques do not allow viewers to precisely look up values when uncertainty is high. Using a visualization technique that nudges viewers toward incorporating uncertainty in their decision-making process is a clever way of indirectly requiring them to use the uncertainty information. On the other hand, for tasks requiring viewers to look up specific values, metaphoric uncertainty can produce worse performance, simply because looking up values can be difficult. We recommend that designers think carefully about the nature of the tasks they are working with and weigh the pros and cons of using metaphoric encodings. Further, as detailed in Hullman et al. [26], researchers need to test uncertainty visualizations with a variety of tasks so that they do not come to incorrect conclusions about the efficacy of a visualization. For example, testing the use of blur with only a point-based look-up task might suggest that blur is a poor visualization choice. However, if a trend or area task were used, blur might prove to be a highly successful technique.

1.3 General Discussion

There are no one-size-fits-all uncertainty visualization approaches, which is why visualization designers must think carefully about each of their design choices or risk adding more confusion to an already difficult decision process. This chapter overviews many of the common uncertainty visualization techniques and the cognitive theory that describes

how and why they function, to help designers think critically about their design choices. We focused on the uncertainty visualization methods and cognitive theories that have received the most support from converging measures (e.g., the practice of testing hypotheses in multiple ways), but there are many approaches not covered in this chapter that will likely prove to be exceptional visualization techniques in the future.

There is no single visualization technique we endorse, but there are some that should be critically considered before employing them. Intervals, such as error bars and the Cone of Uncertainty, can be particularly challenging for viewers. If a designer needs to show an interval, we also recommend displaying information that is more representative, such as a scatterplot, violin plot, gradient plot, ensemble plot, quantile dotplot, or HOP. Just showing an interval alone could lead people to conceptualize the data as categorical.

As alluded to in the prior paragraph, combining various uncertainty visualization approaches may be a way to overcome issues with one technique or get the best of both worlds. For example, each animated draw in a hypothetical outcome plot could leave a trace that slowly builds into a static display such as a gradient plot, or animated draws could be used to help explain the creation of a static technique such as a density plot, error bar, or quantile dotplot. Media outlets such as the New York Times have presented animated dots in a simulation to show inequalities in wealth distribution due to race [76]. More research is needed to understand if and how various uncertainty visualization techniques function together. It is possible that combining techniques is useful in some cases, but new and undocumented issues may arise when approaches are combined.

In closing, we stress the importance of empirically testing each uncertainty visualization approach. As noted in numerous papers [4, 21–23], the way that people reason with uncertainty is non-intuitive, which can be exacerbated when uncertainty information is communicated visually. Evaluating uncertainty visualizations can also be challenging, but it is necessary to ensure that people correctly interpret a display [26]. A recent survey of uncertainty visualization evaluations offers practical guidance on how to test uncertainty visualization techniques [26].

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